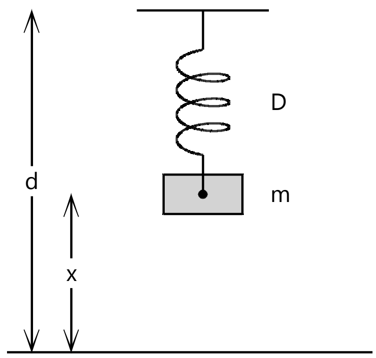
Exercise Sheet 1

Mass-Spring-Pendulum



In this task, a mass-spring pendulum is to be simulated. The mass spring consists of a mass an $m = 0{,}03\,kg$ d a spring with the spring constant $D = 0{,}6\frac{N}{m}$ and a rest length $l_0 = 0{,}2\,m$. At the upper end, the spring is connected with a fixed point and at the lower end it is connected wit with the mass. The fixed point is located at a height of $d = 8\,m$ above the ground. For simplification, it is assumed that the mass is point-shaped. By gravity, the mass is accelerated downwards with an acceleration of $g = 9{,}81\frac{m}{s^2}$.

During the simulation the pendulum swings up and down. The position of the pendulum x is measured from the ground. Variable x describes the distance between soil and mass. The position of the mass is a physical quantity that depends on time. At the beginning, at $t = 0\,s$, the mass $1\,m$ is above the ground. The following applies: $x(0) = 1{,}5\,m$.

Let v be velocity of the mass. The upward speed shall have a positive sign and the downward speed a negative sign. At the beginning, the mass is at rest. It applies: $v(0) = 0\frac{m}{s}$.

Let F be the force acting on the mass and let a be the acceleration applying to the mass. The upward force shall have a positive sign, the downward force a negative sign. Same goes for the acceleration.

Exercise 1

Provide equations for determining the values of F and a using the above mentioned variables $m, D, l_{0,} d, g, x$ and v .

Exercise 2

Several physical variables and formulas have been used to describe the physics of the mass-spring-pendulum. The physical variables and formulas of this physical model shall now be listed in a systematic manner using the following table.

State variables are variables that define the current state of the physical system. For each state variable, enter its name, its physical unit and its initial value at the start of the simulation! To get the simulation running, derivations after time are required for every state variable. For each state variable, enter the variable's name, its physical unit, its initial value and the name of the variable representing its derivation!

Dependent variables are variables whose values depend on the constants and state variables. For each dependent variable there is a formula with which you can determine the value of the dependent variable using the constants and the current values of the state variables. In this example, there are two dependent variables F and a . The formulas for calculating their values were already determined in the first exercise of this exercise sheet. Enter these two variables in the lower part of the table under "dependent variables". For every dependent variable, enter its name, its physical unit, and its formula.

Constants

name	*unit*	*value*

State Variables

name	*unit*	*initial value*	*derivation*

Dependent Variables

name	*unit*	*formula*

Exercise 3

In a next step, the physical system is to be programmed. First create a class called *MassSpringPendulum*. Copy the following program code.

```
import de.physolator.usr.*;
import static java.lang.Math.*;

public class MassSpringPendulum extends PhysicalSystem {

    // This is where the phyisical Variables are to be declared.
    // Examples:
    // @V(unit = "m/s^2")
    // public double g =  9.81;
    // @V(unit = "m", derivative = "v")
    // public double x = 1.5;
    // @V(unit = "m/s", derivative = "a")
    // public double v = 0;
    // @V(unit = "m/s^2")
    // public double a;

    public void f(double t, double h) {
        // This is where the formulas are to be placed.
        // Example:
        // F = - g * m;
    }

    public void initPlotterDescriptors(PlotterParameters r) {
        r.add("x,v", 25, -2, 2);
    }
}
```

The physical variables and formulas from the table of exercise 2 are to be inserted at the points marked with comments. In the comments you will find examples explaining how to do this.

Start with the physical variables. Create a Java variable of type *double* for each physical variable! Put a *@V* annotation in front of each Java variable! In the *@V* annotation, specify the physical unit of the variable! For the state variables in the, also specify the associated derivation of the variable inside the @V annotation.

The method *f* contains the physical formulas that describe the behavior of the physical system. The formulas are represented by assignments. Each assignment assigns a value to a dependent variable according to the formulas from the table of exercise 2.

Explanation of the program code

The *extends PhysicalSystem* specifies that the class *MassSpringPendulum* inherits from the class *PhysicalSystem.* In other words: *MassSpringPendulum* shall be a physical system. The two methods *f* and *initPlotterDescriptors* overwrite the methods inherited from the parent class *PhysicalSystem*. The method *f* contains the physical formulas and the method *initPlotterDescriptors* defines how the function plotter shall work.

The Function Plotter

The method *initPlotterDescriptor* describes how the function plotter shall draw the courses of selected variables over time. The above program code has the effect that a function plotter is attached to the physical system and that the course of the two variables *x* and *v* is displayed in it. The horizontal coordinate axis of the plotter represents time. The last 25 seconds is to be displayed on this axis. The vertical coordinate axis of the plotter represents the values of the variables. On the screen, the vertical coordinate axis shall range from -2 to 2.

Exercise 4

Load the physical system into the Physolator and start the simulation!

Vary the spring constant *D*, the mass *m* and the initial values of *x* and *v* in your program code. Load the modified physical system into the Physolator with the *reload* button and restart the physical system! What influence do these four values *D*, l_0, *m*, *x(0)* and *v(0)* have on the behavior of the physical system?

Exercise 5

When moving through air, the mass experiences flow resistance. The flow resistance is a force directed against the movement. This force causes the pendulum movement to slow down further and further and the pendulum loses more and more energy. One speaks of a damped vibration.

The flow resistance has not yet been taken into account. Extend the existing physical system in such a way that the flow resistance is also considered when calculating the force. Let us suppose the mass be a ball with a radius of $r=0{,}2m$. The following equation describes the flow resistance F_L . Make sure that the flow resistance F_L is always directed in the opposite direction to the direction of movement *v*.

$$F_L = \frac{1}{2} A c_w \rho v^2$$

In this equation equation, $A=r^2\pi$ is the ball's cross sectional area, $c_w=0{,}15$ is the flow resistance coefficient for a spherical body and $\rho=1{,}2041\frac{kg}{m^3}$ is the mass density of air.

Hint

The formula $F_L = \frac{1}{2} A c_w \rho v^2$ always returns a positive value. To ensure that the force always opposes the direction of movement *v*, multiply this value by *-signum(v)*. The signum function maps positive numbers to 1, negative numbers to -1 and 0 to 0. In Java, you find function *signum* inside class *Math.*

Exercise 6

Vary the parameters r , c_w and ρ ! What influence do these parameters have on the behaviour of the physical system?

Exercise 7

Determine the position where the pendulum is in equilibrium, i. e. where the spring force and the gravitational force cancel each other out!

Perform a physical simulation where the mass is initially at equilibrium height, then a simulation where the mass is initially above equilibrium height, and finally a simulation where the mass is initially below equilibrium height! Explain the simulation results!

Exercise 8

The pendulum is moved to the lunar surface. The moon has no atmosphere and the gravitational acceleration on the moon's surface is $g = 1{,}62\frac{m}{s^2}$. How does the pendulum behave on the lunar surface?